NOTE ON PROPOSITION 3.6

Today in class we had the following setup in the last result about rings of fractions. (This result was Prop 3.6 in the textbook.)

Given a ring homomorphism $f : A \longrightarrow B$ and a prime ideal I of A with $I = I^{ec}$, we wanted to show that $I^e \cap f(A \setminus I) = \emptyset$.

Say that we found an element $y \in I^e \cap f(A \setminus I)$. Then there exists $x \in (A \setminus I)$ such that $f(x) = y \in I^e$. But this implies that $x \in f^{-1}(I^e) = I^{ec} = I$ which contradicts the fact that $x \in (A \setminus I)$.