HOMEWORK 4

DUE 29 APRIL 2016

Part I.

1. Show that $R_1 \otimes_{\mathbb{Z}} R_2$ is the coproduct of R_1 and R_2 in the category of commutative rings.

2. (a) Let A be an R-algebra and I an ideal in R. Show that $R/I \otimes_R A \simeq A/J$ as R-algebras, where $J = I^e$ is the extension of the ideal I to an ideal of A (i.e., the ideal of A generated by the image of I via the structure homomorphism).

(b) If A is an R-algebra and I an ideal of $R[X]$, show that $A \otimes_R (R[X]/I) \simeq A[X]/J$, where J is the ideal of $A[X]$ generated by the image of I, i.e., the extension of I to $A[X]$ via the map $R[X] \longrightarrow A[X]$ induced by the structure homomorphism of A.

3. Show that

(a) $R[X] \otimes_R R[Y] \simeq R[X, Y]$ as R-algebras.

(b) $R/I \otimes_R R/J \simeq R/(I + J)$ for any two ideals I, J of R.

Part II. From Atiyah-MacDonald

Chapter 2: 3, 4, 6, 17, 20

Bonus.

B1. Let

$$R = \{f : [0, 1] \longrightarrow \mathbb{R}; f \text{ is continuous and } f(0) = f(1)\}$$

and

$$M = \{g : [0, 1] \longrightarrow \mathbb{R}; g \text{ is continuous and } g(0) = -g(1)\} .$$

Then R is a commutative ring under addition and multiplication of functions and M is an R-module. Is M free as an R-module? Is it projective?

Atiyah & Macdonald, Chapter 2: 7, 21