1. Compute the double integral

$$\iint_D (x^2 + y^2)\, dA$$

where D is the annulus between circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 1/4$.

Solution. The domain D is described by the inequalities

$$0 \le \theta \le 2\pi, \quad \frac{1}{2} \le r \le 1.$$

The function $f(x, y) = x^2 + y^2$ in polar coordinates is

$$f(x, y) = x^2 + y^2 = (r\cos\theta)^2 + (r\sin\theta)^2 = r^2.$$

Using change of variables in polar coordinates gives

$$\begin{aligned}
\iint_D (x^2 + y^2)\, dA &= \int_0^{2\pi} \int_{\frac{1}{2}}^{1} (r^2) r \, dr \, d\theta \\
&= \int_0^{2\pi} \left. \frac{r^4}{4} \right|_{r=\frac{1}{2}}^{1} d\theta \\
&= \int_0^{2\pi} \frac{15}{64}\, d\theta \\
&= \frac{15}{64}(2\pi) \\
&= \frac{15}{32}\pi.
\end{aligned}$$

□