Quiz 4, Section A04 Solutions

Compute the double integral

$$\int\int_D x - 1 \ dA$$

where D is the region $0 \le x \le 2 - y \le 2$

Solution: Since $x \le 2 - y \le 2$, we can subtract 2 from both sides to get $x - 2 \le -y \le 0$, and then we can multiply by -1 to get $2 - x \ge y \ge 0$. So the integral is:

$$
\begin{aligned}
\int\int_D x - 1 \ dA &= \int_0^2 \int_0^{2-x} x - 1 \ dydx \\
&= \int_0^2 \left(\frac{y^2}{2} - y \right) \Big|_0^{2-x} dx \\
&= \int_0^2 \frac{(2-x)^2}{2} - (2-x) \ dx \\
&= \int_0^2 \frac{4 - 4x + x^2}{2} - 2 + x \ dx \\
&= \int_0^2 2 - 2x + \frac{x^2}{2} - 2 + x \ dx \\
&= \int_0^2 -x + \frac{x^2}{2} \ dx \\
&= \left(\frac{-x^2}{2} + \frac{x^3}{6} \right) \Big|_0^2 \\
&= \frac{-4}{2} + \frac{8}{6} \\
&= -2 + \frac{4}{3} \\
&= \frac{-2}{3}
\end{aligned}
$$