

Quiz 4, Section A03 Solutions

Compute the double integral

$$\iint_D x + 1 \, dA$$

where  $D$  is the region  $0 \leq 1 - x \leq y \leq 1$ .

*Solution:* Since  $0 \leq 1 - x \leq y$ , we can subtract 1 to get  $-1 \leq -x \leq y - 1$ , and then multiply by  $-1$  to get  $1 \geq x \geq 1 - y$ . so we can compute the integral:

$$\begin{aligned} \iint_D x + 1 \, dA &= \int_0^1 \int_{1-y}^1 x + 1 \, dx dy \\ &= \int_0^1 \left( \frac{x^2}{2} + x \right) \Big|_{1-y}^1 dy \\ &= \int_0^1 \left( \frac{1}{2} + 1 \right) - \left( \frac{(1-y)^2}{2} + 1 - y \right) dy \\ &= \int_0^1 \frac{1}{2} - \frac{(1-y)^2}{2} + y \, dy \\ &= \int_0^1 \frac{1}{2} - \frac{1 - 2y + y^2}{2} + y \, dy \\ &= \int_0^1 \frac{1}{2} - \frac{1}{2} + y - \frac{y^2}{2} + y \, dy \\ &= \int_0^1 2y - \frac{y^2}{2} \, dy \\ &= \left( y^2 - \frac{y^3}{6} \right) \Big|_0^1 \\ &= 1 - \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$