

Quiz 4, Section A02 Solutions

Compute the double integral

$$\iint_D 2xy \, dA$$

where D is the region $0 \leq x \leq 1 - y \leq 1$.

Solution: Since $0 \leq 1 - y \leq 1$, we know $0 \leq y \leq 1$. So we can write the integral:

$$\begin{aligned} \iint_D 2xy \, dA &= \int_0^1 \int_0^{1-y} 2xy \, dx dy \\ &= \int_0^1 x^2 y \Big|_0^{1-y} dy \\ &= \int_0^1 (1-y)^2 y \, dy \\ &= \int_0^1 y - 2y^2 + y^3 \, dy \\ &= \left(\frac{y^2}{2} - \frac{2y^3}{3} + \frac{y^4}{4} \right) \Big|_0^1 \\ &= \frac{1}{2} - \frac{2}{3} + \frac{1}{4} \\ &= \frac{1}{12} \end{aligned}$$