

Solutions for Quiz 2, Section A01

Find the angle P of the space triangle PQR formed by the points

$$P = (-1, 1, 0), \quad Q = (2, 1, 1), \quad R = (1, 0, 2)$$

Solution: The angle θ at point P is the angle between the vectors \overrightarrow{PQ} and \overrightarrow{PR} . These vectors are:

$$\overrightarrow{PR} = \langle 1 - (-1), 0 - 1, 2 - 0 \rangle = \langle 2, -1, 2 \rangle$$

$$\overrightarrow{PQ} = \langle 2 - (-1), 1 - 1, 1 - 0 \rangle = \langle 3, 0, 1 \rangle$$

So we can compute:

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 6 + 0 + 2 = 8$$

$$\|\overrightarrow{PQ}\| = \sqrt{9 + 0 + 1} = \sqrt{10}$$

$$\|\overrightarrow{PR}\| = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Therefore:

$$\cos(\theta) = \frac{8}{3\sqrt{10}}$$

So $\theta = \arccos\left(\frac{8}{3\sqrt{10}}\right)$, where θ is chosen to be between 0 and π since it is an interior angle of a triangle.