Solutions for Quiz 1, Section A01

Let $A = (a, 1)$, $B = (2, 2)$, $C = (4, -1)$, and $D = (0, b)$. Find the values of a and b for which $ABCD$ is a parallelogram.

Solution: For $ABCD$ to be a parallelogram, $\overrightarrow{AB}$ must equal $\overrightarrow{DC}$. By plugging in the values of the vectors we can solve:

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{DC} \\ \langle 2 - a, 2 - 1 \rangle &= \langle 4 - 0, -1 - b \rangle \\ \langle 2 - a, 1 \rangle &= \langle 4, -1 - b \rangle \end{aligned}$$

So $2 - a = 4$ and $-1 - b = 1$, so $a = -2$ and $b = -2$.

Another solution method: For $ABCD$ to be a parallelogram, the intersection of the diagonals must also be the midpoint of the diagonals. This means that the midpoint of $\overline{AB}$ is the same as the midpoint of $\overline{BD}$. So:

$$\begin{aligned} \left(\frac{4 + a}{2}, \frac{-1 + 1}{2} \right) &= \left(\frac{2 + 0}{2}, \frac{2 + b}{2} \right) \\ \left(\frac{4 + a}{2}, 0 \right) &= \left(1, \frac{2 + b}{2} \right) \end{aligned}$$

Looking at the x-coordinates:

$$\begin{aligned} \frac{4 + a}{2} &= 1 \\ 4 + a &= 2 \\ a &= -2 \end{aligned}$$

Looking at the y-coordinates:

$$\begin{aligned} 0 &= \frac{2 + b}{2} \\ 0 &= 2 + b \\ b &= -2 \end{aligned}$$