PRACTICE PROBLEMS

DISCLAIMER: The actual midterm questions may have nothing to do with the ones below.

Justify all your work.

1. Decompose into primes in $\mathbb{Z}[i]$ the following gaussian integers.
   (a) 225;
   (b) $9+3i$.
2. Prove that a gaussian integer is divisible by $\alpha = 1+i$ if and only if its norm is even.
3. Find a positive integer that can be written as a sum of squares in 3 ways. *It is not enough to exhibit the 3 ways to write it as a square, you have to explain how you found it.*
4. Find all the integer solutions (or show that none exist) to the diophantine equation
$$x^2 - 25y^2 = 39.$$
5. Compute the continued fraction of the following numbers.
   (a) $\dfrac{1 \pm \sqrt{3}}{2}$:
   (b) $\sqrt{6}$.
6. Represent as $\frac{r+s\sqrt{d}}{t}$ the following continued fractions.
   (a) $[3, \bar{5}]$;
   (b) $[1, 3, \bar{5}]$.
7. (a) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 7y^2 = -1$.
   (b) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 7y^2 = 1$.
8. **(Extra credit)** If p is a prime, show that Pell's equation
$$x^2 - py^2 = -1$$
has integer solutions if and only if $p = 2$ or $p \equiv 1 \pmod 4$.
Hint: Consider $a+1$ and $a-1$ where (a, b) is a solution to $a^2 - pb^2 = 1$.