PRACTICE PROBLEMS

DISCLAIMER: The actual final exam questions may have nothing to do with the ones below.

Justify all your work.

1. Find the 7th root of 31 modulo 33.
2. Find the greatest common divisor of the gaussian integers $17 + 71i$ and $6 - 69i$.
3. Determine which of the following numbers can this be written as a sum of two squares. You do not need to exhibit a sum of squares equal to each number, but you do need to give your reasoning.

$$245; \quad 1245; \qquad 900000000000; \quad 3333333333.$$

4. Compute the continued fraction of the following numbers.
   (a) $\dfrac{1 \pm \sqrt{3}}{2}$:
   (b) $\sqrt{6}$.
5. Represent as $\frac{r+s\sqrt{d}}{t}$ the following continued fractions.
   (a) $[2, \bar{5}]$;
   (b) $[-1, 2, \bar{5}]$.
6. Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 6y^2 = -1$.
7. Compute the following Jacobi symbols.
   (a) $\left(\dfrac{977}{1001}\right)$
   (b) $\left(\dfrac{248}{563}\right)$
8. (a) Use quadratic reciprocity to determine the congruence classes in $(\mathbb{Z}/84\mathbb{Z})^\times$ with $\left(\dfrac{-21}{p}\right) = 1$.
   (b) Determine for which congruences classes modulo 84, the prime $p \mid a^2 + 21b^2$ for some relatively prime integers a, b. The answer should be a list, e.g. $p \mid a^2 + 21b^2$ for a, b relatively prime if and only if $p \equiv \ldots \pmod{84}$.
9. Find all the integer solutions (or show that none exist) to the diophantine equation

$$x^2 + 3y^2 = z^2.$$

10. Prove that a prime number p can be written as $p = a^2 + 2b^2$ for some integers a, b if and only if $p = 2$ or $p \equiv 1$ or $3 \pmod 8$.

Note: *This practice test has one extra problem with respect to what you can expect on the final exam.*