MATH 20E — Lecture B
Final Exam — June 12, 2013

Name: ______________________

PID: ______________________

TA: ______________________

Section #: __________ Section time: ______________

There are 12 pages and 7 questions, for a total of 150 points.

No books, no notes except the one 8.5×11in sheet, no electronic devices except the handheld calculator.
Calculator policy: You may use any handheld calculator (but not a computer, PDA or any device with network capability; no device that has internal or external memory or hard-drive on which material can be stored) during exams; however, you will be expected to show all relevant computational steps: No credit will be given for unsupported answers gotten directly from a calculator.

Please turn off all electronic devices (except for the handheld calculator).
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.
Please write your name, PID and Section on all pages.

Good luck! ☺

Half-angle formulas:

$$\cos^2 t = \frac{1+\cos(2t)}{2}, \quad \sin^2 t = \frac{1-\cos(2t)}{2}, \quad \sin t \cos t = \frac{\sin(2t)}{2}$$

Question:	1	2	3	4	5	6	7	Total
Points:	10	10	10	25	40	25	30	150
Score:								

1. Consider the function $f(x, y, z) = x\sqrt{z + y^2} + \dfrac{y}{x}$.

   (a) (5 points) Compute the gradient of f at the point $P = (1, 3, 2)$.

   (b) (5 points) If starting at the point P a small change is to be made in only *one* of the variables, which one would produce the largest change (in absolute value) in f?

2. (10 points) Let R be the region in the plane bounded by the curves

$$xy^4 = 4, xy^4 = 9, \frac{x}{y} = 1, \text{ and } \frac{x}{y} = 2.$$

Evaluate $\iint_R 2x^2y - x^2y^6 dxdy$. (This type of question arises in thermodynamics.)
Hint: change variables and do not forget the absolute value!

3. (10 points) Find the flux of $\vec{F} = z\hat{\mathbf{i}} + \hat{\mathbf{j}} - x^2y\hat{\mathbf{k}}$ across the surface S parametrized by

$$x = 2uv, y = u, z = 2v$$

with $0 \leq u, v \leq 1$. The choice of orientation is left to you.

4. (a) (5 points) Prove that the vector field

$$\vec{F} = (2xyz - yz^2)\hat{\mathbf{i}} + (x^2z - xz^2 + 2y)\hat{\mathbf{j}} + (x^2y - 2xyz)\hat{\mathbf{k}}$$

is conservative.

(b) (15 points) Using a *line integral*, find a potential function for $\vec{F}$. Show all your work, even if you can do it mentally.

(c) (5 points) Find the work done by $\vec{F}$ along the straight line from $(1, 1, 4)$ to $(2, -1, -2)$ (in that order) using as little computation as possible.

5. Consider the triangular region R with vertices at $(0,0)$, $(0,1)$ and $(-1,1)$. Let C be the boundary of R oriented counterclockwise and $\vec{F} = x(\hat{\mathbf{j}} - \hat{\mathbf{i}})$.

   (a) (2 points) Sketch C and R.

   (b) (15 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ by converting it into a *double integral* over R. (Set up, and then it is ok to use a shortcut.)

   (c) (3 points) Is $\vec{F}$ a gradient field?

(d) (10 points) Compute the flux integral $\int_C \vec{F} \cdot \hat{\mathbf{n}} ds$ directly as a line integral. Use the given orientation.

(e) (10 points) Compute the flux integral $\int_C \vec{F} \cdot \hat{\mathbf{n}} ds$ using the appropriate *double integral.* (Set up, and then it is ok to use a shortcut.)

Reality check: The answers to (c) and (d) should coincide.

6. In this problem S is the surface given by the quarter of the cylinder of radius 3 centered around the z-axis with $x, y \geq 0$ and $0 \leq z \leq 4$. Consider the vector field $\vec{F} = z\hat{\mathbf{j}}$.

   (a) (5 points) Sketch the surface S and the vector field $\vec{F}$.

   (b) (10 points) Compute the flux of $\vec{F}$ through S. Specify which orientation you are using on S.

(c) (5 points) Let W be the solid in the first octant (the region with $x, y, z \geq 0$) given by the interior of the quarter cylinder defined above. Use the divergence theorem to compute the flux of $\vec{F}$ out of the region W.

(d) (5 points) The boundary surface of W consists of S together with four other faces. What is the flux outward through these four faces and why?

7. Let $\vec{F} = (z, y, -x)$. Let S be the portion of the surface of the paraboloid $z = 1 - x^2 - y^2$ which lies in the first octant (i.e. $x, y, z \geq 0$). Consider the closed curve $C = C_1 + C_2 + C_3$ where C_1, C_2 and C_3 are the three curves formed by intersecting S with the xy-plane, the yz-plane and the xz-plane respectively. (Note that C is the boundary of S.) Orient C so that it is traversed counterclockwise when seen from above in the first octant.

   (a) (5 points) Sketch S and C.

   (b) (15 points) Use Stokes' theorem to compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ by using the *surface integral* over the capping surface S.

(c) (10 points) Set up and evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ directly by parametrizing each piece of the curve C and then adding up the three line integrals.

Reality check: The answers to (b) and (c) should coincide.