MATH 200B Name: ______________________

MIDTERM EXAM

24 FEBRUARY 2014

Do as many of the problems as well as you can; you are not necessarily expected to finish all problems. You may quote results proved in class or in the textbook, but not if they trivialize the problem. Also, avoid quoting results proved only in homework exercises. All rings are unital.

1. Find $\mathbb{Z}/12\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}/6\mathbb{Z}$.

2. Find all the isomorphism classes of abelian groups with 100 elements. List both their primary decomposition and their elementary divisors decomposition.

3. Show that a projective module is flat (over a commutative ring).

4. Let R be a nonzero commutative ring. Let $M = R^m = \bigoplus_{i=1}^{m} R$ and $N = R^n$ be R-modules. Let $\phi : M \longrightarrow N$ be an R-module homomorphism.

(a) If ϕ is an isomorphism, show that $m = n$. (Hint: Consider a maximal ideal $\mathfrak{m}$ of R.)

(b) If ϕ is surjective, show that $m \geq n$.

(c) (Bonus) If ϕ is injective, is it necessarily the case that $m \leq n$?

5. Let $n \geq 1$ and consider the $\mathbb{C}[X]$-module $M = \mathbb{C}[X]/(X - \alpha)^n$. Show that there exists a $\mathbb{C}$-basis for M such that the matrix of the corresponding linear operator is a Jordan block.