HOMEWORK 4

DUE 7 FEBRUARY 2014

1. Let
$$R = \{f : [0,1] \longrightarrow \mathbb{R}; f \text{ is continuous and } f(0) = f(1)\}$$
and
$$M = \{g : [0,1] \longrightarrow \mathbb{R}; g \text{ is continuous and } g(0) = -g(1)\}.$$
Then R is a commutative ring under addition and multiplication of functions and M is an R-module. Is M free as an R-module? Is it projective?

2. This is a continuation of problem 2 on HW3. Let G be a finite group. For any G-module M, let
$$C(M) = \{ \text{ functions } f : G \longrightarrow M\},$$
$$Z(M) = \{f \in C(M);\ f(\sigma\tau) = \sigma f(\tau) + f(\sigma)\ \forall \sigma, \tau \in G\},$$

$$B(M) = \{f \in C(M);\ \exists m \in M \text{ so that } f(\sigma) = \sigma m - m\, \forall \sigma \in G\}.$$

(a) Check that $B(M)$ and $Z(M)$ are abelian groups and that $B(M) \subseteq Z(M)$.

(b) Let $H(M) = Z(M)/B(M)$. If G acts trivially on M, show that as abelian groups $H(M) \cong \mathrm{Hom}(G, M)$ where $\mathrm{Hom}(G, M)$ is the set of all group homomorphisms $G \longrightarrow M$.

(c) For any short exact sequence of G-modules
$$0 \longrightarrow M \longrightarrow N \longrightarrow P \longrightarrow 0,$$
show that there is an exact sequence
$$0 \longrightarrow M^G \longrightarrow N^G \longrightarrow P^G \longrightarrow H(M) \longrightarrow H(N) \longrightarrow H(P).$$

From Dummit and Foote: section **10.2** problem 8; section **10.3** problems 4, 5, 9 (read definition of cyclic module on page 351); section **10.5** problems 3, 4, 10 (except for flatness in part c), 11.