

MIDTERM

7 November 2013

No calculators, no books, no notes.

There are 6 questions, for a total of 100 points.

For credit you need to write clearly and fully justify everything you write down. If you use theorems proved in class, or in the book, or elsewhere, in the course of a proof, make clear what result you are using. We will grade only what is written on the page and marked as part of the solution (e.g. not crossed out).

Do not get hung up on one problem. Make sure you get to work on all of them.

1. (10 points) Compute $d(45000)$.
2. (20 points) How many primitive roots are there mod 81?
3. (15 points) Factor the polynomial $X^3 + 3X - 3$ into linear factors (mod 7) or show that it is irreducible (mod 7).
4. (20 points) Show that $(n^3 - n) \equiv 0 \pmod{6}$ for any integer n .
5. (15 points) Show that if n is an odd integer and a is an integer, then

$$a^n \equiv a \pmod{3}.$$

6. (20 points) The following multiplication was performed correct, but unfortunately the printer inserted an x in place of a digit in the answer:

$$172\,195 \cdot 572\,167 = 98\,524\,x96\,565.$$

Determine x without redoing the multiplication. Justify in detail how you found x .

Question:	1	2	3	4	5	6	Total
Points:	10	20	15	20	15	20	100
Score:							

Good luck! ☺