MIDTERM

7 November 2013

No calculators, no books, no notes.
There are 5 questions, for a total of 100 points.
For credit you need to write clearly and fully justify everything you write down. If you use theorems proved in class, or in the book, or elsewhere, in the course of a proof, make clear what result you are using. We will grade only what is written on the page and marked as part of the solution (e.g. not crossed out).
Do not get hung up on one problem. Make sure you get to work on all of them.

1. (10 points) Compute $\sigma(300)$.

2. (20 points) Find three consecutive positive integers such that the first is divisible by 7, the second is divisible by 9, and the third is divisible by 11.

3. (15 points) Factor the polynomial X^2+3X-3 into linear factors $\pmod{11}$ or show that it is irreducible $\pmod{11}$.

4. (15 points) Show that $n(n-1) \equiv 0 \pmod 2$ for any integer n.

5. (a) (10 points) Given that 3 is a primitive root mod 31, show that $3^5, 3^{10}, 3^{15}, 3^{20}, 3^{25}$ and 3^{30} are the six distinct roots $\pmod{31}$ of the equation

$$x^6 \equiv 1 \pmod{31}.$$

   (b) (10 points) Show that

$$X^6 - 1 = (X-1)(X+1)(X^2-X+1)(X^2+X+1).$$

   (c) (5 points) Show that $3^5, 3^{10}, 3^{15}, 3^{20}, 3^{25}$ and 3^{30} are the six distinct roots $\pmod{31}$ of the equation

$$(x-1)(x+1)(x^2-x+1)(x^2+x+1) \equiv 0 \pmod{31}. \quad (1)$$

   (d) (15 points) Without substituting these solutions into equation (1), find out which solutions goes with which factor.

Question:	1	2	3	4	5	Total
Points:	10	20	15	15	40	100
Score:						

Good luck! ☺