

## MIDTERM

7 November 2013

**No calculators, no books, no notes.**

There are 6 questions, for a total of 100 points.

For credit you need to write clearly and fully justify everything you write down. If you use theorems proved in class, or in the book, or elsewhere, in the course of a proof, make clear what result you are using. We will grade only what is written on the page and marked as part of the solution (e.g. not crossed out).

Do not get hung up on one problem. Make sure you get to work on all of them.

- (10 points) Compute  $\text{ord}_7(5)$ .
- (20 points) A multiplication has been performed incorrectly, but the answer is correct  $(\text{mod } 9)$ ,  $(\text{mod } 10)$  and  $(\text{mod } 11)$ . What is the closest that the incorrect result can be to the correct result?
- (15 points) Factor the polynomial  $X^3 - 4X + 2$  into linear factors  $(\text{mod } 5)$  or show that it is irreducible  $(\text{mod } 5)$ .
- (20 points) Show that  $(n^3 - n) \equiv 0 \pmod{6}$  for any integer  $n$ .
- (20 points) Let  $a = 13259768$ . Prove in detail that  $p \equiv 1 \pmod{8}$  for every prime factor  $p$  of  $(a^4 + 1)$ .
- (15 points) Show that  $a^2 + b^2 \not\equiv -1 \pmod{4}$  for any integers  $a, b$ .

Question:	1	2	3	4	5	6	Total
Points:	10	20	15	20	20	15	100
Score:							

Good luck! ☺