# HOMEWORK 6

DUE 21 FEBRUARY 2013

**1.** Find all positive integer solutions to $x^2 + 12 = y^4$.

**2.** Find all positive integer solutions to $x^3 + y^3 = 20$.

**3.** For each of the following gaussian integers, determine if $\alpha \mid \beta$. Justify your answer.
(a) $\alpha = 2, \beta = 3 + i$;
(b) $\alpha = 1 + i, \beta = 3 + i$;
(c) $\alpha = 1 + i, \beta = 1599 + 2478i$;
(d) $\alpha = 3 + 4i, \beta = 3 - 4i$;
(e) $\alpha = 1 + 4i, \beta = 3 - 14i$.

**4.** Determine if $1 + 4i$, $2 + 3i$ and $9 - 7i$ are prime elements of $\mathbb{Z}[i]$. Justify your answer.

**5.** Find the prime factorization of $2, 3, 5$ and $25$ in $\mathbb{Z}[i]$.

**6.** Find a greatest common divisor for the following pairs of gaussian integers.
(a) $\alpha = 2, \beta = 3 + i$;
(b) $\alpha = 2, \beta = 3 + 3i$;
(c) $\alpha = 1 + i, \beta = 1599 + 2478i$.

**7.** Prove or disprove and salvage if possible the following statement: if $\alpha$ and $\beta$ are relatively prime gaussian integers with $\alpha\beta = \gamma^2$ for some $\gamma \in \mathbb{Z}[i]$, then $\alpha$ and $\beta$ are both squares.