

HOMEWORK 5

DUE 14 FEBRUARY 2013

1. Find all integer solutions to $x^2 + 3y^2 = z^2$ with $x > 0, y > 0, z > 0$.
2. Find all integer solutions to $x^2 + py^2 = z^2$ with $x > 0, y > 0, z > 0$, where p is a fixed odd prime.
3. Find all integer solutions to $x^2 + y^2 = 4z^2$ with $x > 0, y > 0, z > 0$ and $(x, y, z) = 1$.
4. Find all positive integer solutions to $x^3 - y^3 = 19$.
5. Find all integer solutions to $x^2 - dy^2 = 1$ when d is a perfect square.
6. If x, y, z are integers with $x^3 + y^3 + z^3 = 0$, show that
$$(x + y + z)^3 = 3(x + y)(y + z)(x + z).$$
Use this relation to show that 3 must divide one of the integers x, y, z .
7. Show that the equation
$$x^3 + y^3 = z^3$$
has no integer solutions with $3 \nmid xyz$.