HOMEWORK 3

DUE 31 JANUARY 2013

1. Determine whether 888 is a quadratic residue or nonresidue modulo the prime 1999 using exclusively the Legendre symbol.

2. Determine whether 888 is a quadratic residue or nonresidue modulo 1999 by factoring $888 = 2 \cdot 4 \cdot 111$ and using Jacobi symbols.

3. Determine whether -104 is a quadratic residue or nonresidue modulo the prime 997.

4. Use quadratic reciprocity to determine the congruence classes of primes $p \neq 2, 7$ in $(\mathbb{Z}/28\mathbb{Z})^\times$ with $\left(\frac{-7}{p}\right) = 1$.

5. Use quadratic reciprocity to determine the congruence classes of primes $p \neq 2, 3, 7$ in $(\mathbb{Z}/84\mathbb{Z})^\times$ with $\left(\frac{-21}{p}\right) = 1$.

6. Compute $\left(\frac{3}{41}\right)$ and $\left(\frac{3}{47}\right)$.

7. We know that if m is an odd positive integer and $a, b \in \mathbb{Z}$ with $a \equiv b \pmod{m}$ then $\left(\frac{a}{m}\right) = \left(\frac{b}{m}\right)$. Prove or disprove the following statements.

(a) If m, n are positive odd integers and $a \in \mathbb{Z}$ such that $m \equiv n \pmod{a}$ then $\left(\frac{a}{m}\right) = \left(\frac{a}{n}\right)$.

(b) If m, n, a are positive odd integers and $m \equiv n \pmod{a}$ then $\left(\frac{a}{m}\right) = \left(\frac{a}{n}\right)$.

(c) If m, n, a are positive odd distinct integers and $m \equiv n \pmod{a}$ then $\left(\frac{a}{m}\right) = \left(\frac{a}{n}\right)$.