

HOMEWORK 2

DUE 17 APRIL 2013

1. Compute the Galois group $G = \text{Gal}(\mathbb{F}_{3^{16}}/\mathbb{F}_{3^2})$ where \mathbb{F}_q denotes the finite field with q elements. *Hint: Frobenius.*
2. With the notation from the previous problem, find all the subgroups of G .
3. With the notation from the previous two problems, for each subgroup H of G find the fixed field of H .
4. Prove that the splitting field of $X^3 - 2$ over \mathbb{Q} is $K = \mathbb{Q}(\sqrt[3]{2}, \omega)$ where $\omega = \frac{-1+i\sqrt{3}}{2}$ the third root of unity.
5. Compute the Galois group $G = \text{Gal}(\mathbb{Q}(\sqrt[3]{2}, \omega)/\mathbb{Q})$.
6. Find all the subgroups of $G = \text{Gal}(\mathbb{Q}(\sqrt[3]{2}, \omega)/\mathbb{Q})$.
7. With the notation from the previous problem, for each subgroup H of G find the fixed field of H .