

PRACTICE PROBLEMS

DISCLAIMER: The actual midterm questions may have nothing to do with the ones below.

1. Compute the continued fraction of the following numbers.

(a) $\frac{1 \pm \sqrt{3}}{2}$

(b) $\sqrt{6}$

2. Represent as $\frac{r+s\sqrt{d}}{t}$ the following continued fractions.

(a) $[3, \bar{5}]$

(b) $[1, 3, \bar{5}]$

3. (a) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 7y^2 = -1$.

(b) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 7y^2 = 1$.

4. Compute the following Legendre symbols.

$$\left(\frac{3}{11}\right) \quad \left(\frac{11}{3}\right) \quad \left(\frac{22}{3}\right) \quad \left(\frac{22}{11}\right)$$

5. If p is a prime, show that Pell's equation

$$x^2 - py^2 = -1$$

has integer solutions if and only if $p = 2$ or $p \equiv 1 \pmod{4}$.

Hint: Consider $a + 1$ and $a - 1$ where (a, b) is a solution to $a^2 - pb^2 = 1$.