HOMEWORK 7

DUE 6 MARCH 2012

1. Show that equivalence and proper equivalence of bqf's are equivalence relations.

2. Show that improper equivalence is not an equivalence relation.

3. Show that equivalent forms represent the same numbers. Show that the same holds for proper representations. Here you need to show that, given $m \in \mathbb{Z}$, the equation $f(x, y) = m$ has the same number of solutions in integers for all the forms in the same equivalence class. Similarly for proper representations.

4. Show that any form equivalent to a primitive forms is itself primitive. *Hint: Use the previous exercise.*

5. Suppose $f(x, y)$ is a bqf with discriminant D and $g(x, y)$ is a bqf with discriminant D'. Suppose further that

$$f(x, y) = g(\alpha x + \beta y, \gamma x + \delta y) \text{ for some } \alpha, \beta, \gamma, \delta \in \mathbb{Z}.$$

Show that

$$D = (\alpha\delta - \beta\gamma)^2 D'.$$

6. Prove that if two bqf's are equivalent, they have the same determinant. *Hint: Use the previous exercise.*

7. Prove that if $f(x, y) = ax^2 + bxy + cy^2$ is a primitive, positive definite bqf with $|b| \le a \le c$, then

$$f(x, y) \ge (a - |b| + c) \min x^2, y^2.$$

Did you use the fact that $|b| \le a \le c$ in the proof? Was it essential? If so, find a counterexample.

8. **Extra credit** Prove a version of (9.4) from the uniqueness step of Theorem 9.13 that holds in the exceptional cases $|b| = a$ or $c = a$. Use this to complete the uniqueness proof in Theorem 9.13. *Hint: you can show that the $f(x, y)$ and $g(x, y)$ have to be $ax^2 \pm bxy + cy^2$ and then the restriction $b \ge 0$ implies they must be equal.*