HOMEWORK 2

DUE 31 JANUARY 2012

1. Suppose $d = a^2$ is a perfect square. Find all the integer solutions of the Fermat-Pell equation
$$x^2 - dy^2 = 1.$$

2. The number
$$\gamma = \frac{1 + \sqrt{5}}{2}$$
is called the *golden ratio*. For each $0 \le y \le 20$ find the integer x making $|x - y\gamma|$ as small as possible. Which rational number x/y with $1 \le y \le 20$ most closely approximates γ? Feel free to use a computer or calculator.

3. Find a solution to
$$x^2 - 41y^2 = -1$$
in positive integers by plugging in $y = 1, 2, \ldots$ (you won't have to go very far). Use your answer to find a solution to
$$x^2 - 41y^2 = 1.$$

4. Find a solution to
$$x^2 - 11y^2 = 1.$$

For each of the following equations, either find a solution (x, y) in positive integers or explain why no such solution can exist.

5. $x^2 - 11y^2 = 7$.

6. $x^2 - 11y^2 = 433$.

7. $x^2 - 11y^2 = 3$.