# PRACTICE PROBLEMS

**DISCLAIMER: The actual exam questions may have nothing to do with the ones below.**

1. Find all the integer solutions to the following diophantine equations or show that no such solutions exist.
   (a) $x^2 + 9 = y^4$;
   (b) $x^2 - 7y^2 = 3z^2$;
   (c) $x^2 + 11y^2 = z^2$;
   (d) $x^4 + y^4 = z^4$.
2. Compute the continued fraction of the following numbers.
   (a) $\dfrac{1 \pm \sqrt{5}}{2}$
   (b) $\sqrt{15}$
3. Represent as $\frac{r+s\sqrt{d}}{t}$ the following continued fractions.
   (a) $[-2, \overline{4}]$
   (b) $[1, 3, \overline{4, 5}]$
4. (a) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 5y^2 = -1$.
   (b) Find *all* integer solutions, or prove that no such solutions exist, to $x^2 - 5y^2 = 1$.
5. Compute the following Legendre symbols or explain why they do not make sense.

$$\left(\frac{-300}{11}\right) \quad \left(\frac{11}{300}\right) \quad \left(\frac{229}{13}\right) \quad \left(\frac{231}{91}\right)$$

6. Use the homomorphism $\chi_D$ and the results of Section 9 in the notes to formulate and prove the results equivalent to (9.7) on page 55 of the notes for the following discriminants.
   (a) $D = -31$
   (b) $D = -52$.