EXTRA PROBLEMS

DUE 3 FEBRUARY 2012 IN CLASS

I gave in to the request of assigning more extra credit. The bad news is that this whole set is worth just 3 points (i.e. only as much as *one* regular homework problem).

These problems are not directly related what we covered in class, but they are cool! And they are related to algebraic geometry. In fact, you get brownie points for figuring out what fact in algebraic geometry they hint at.

Let $P_1, \ldots, P_{n+1}$ be distinct points in the plane.

1. For $n = 1$, prove that there exists a conic that passes through P_1 and not P_2.

2. For $n = 2$ prove that there exists a conic that passes through P_1, P_2 and not P_3.

3. How far can you go before it stops being true? That is, what is the highest n with the property that you can always find a conic passing through n points that avoids the $(n+1)$-st point?

And now do the same for cubics.

4. For $n = 1$ prove that there exists a cubic that passes through P_1 and not P_2.

5. For $n = 2$ prove that there exists a cubic that passes through P_1, P_2 and not P_3.

6. For $n = 3$ prove that there exists a cubic that passes through P_1, P_2, P_3 and not P_4.

7. How far can you go before it stops being true? That is, what is the highest n with the property that you can always find a cubic passing through n points that avoids the $(n+1)$-st point?