MATH 20C - Monday, October 18, 2010: first midterm

MATH 20C Lecture 11 - Wednesday, October 20, 2010

Functions of several variables

Recall: for a function of 1 variable, we can plot its graph, and the derivative is the slope of the tangent line to the graph. Plotting graphs of functions of 2 variables: examples $z = -y$, $z = 1 - x^2 - y^2$, using slices by the coordinate planes. (derived carefully). Contour map: level curves $f(x, y) = c$. Amounts to slicing the graph by horizontal planes $z = c$.

Showed 2 examples from "real life": a topographical map, and a temperature map, then did the examples $z = -y$ and $z = 1 - x^2 - y^2$. Showed more examples of computer plots ($z = y^2 - x^2$, and another one).
Contour map gives some qualitative info about how f varies when we change x, y. (shown an example where increasing x leads f to increase).

MATH 20C Lecture 12 - Friday, October 22, 2010

Reviewed countour maps and level curves.

Limits

By substitution:

$$\lim_{(x,y)\to(0,0)} \frac{3x^2 + 2y - 5\cos(4(x+y))}{xy - e^{x-y}} = \frac{0+0-5}{0-1} = 5$$

Disclaimer: limits do not always exist!
For instance, take $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$. Direct substitution does not work. Drawn contour map. We see that a bunch of level curves intersect at $(0, 0)$, so the limit does not exist.

On the other hand $\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{(x+y)} = 1$.

Partial derivatives

$f_x = \dfrac{\partial f}{\partial x} = \lim\limits_{\Delta x\to 0} \dfrac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$; same for f_y.

Geometric interpretation: f_x, f_y are slopes of tangent lines of vertical slices of the graph of f (fixing $y = y_0$; fixing $x = x_0$).
How to compute: treat x as variable, y as constant.
Example: $f(x, y) = x^3y + y^2$, then $f_x = 3x^2y, f_y = x^3 + 2y$.

Another example: $g(x, y) = \cos(x^3y + y^2)$.

Use chain rule (version I)

$$\boxed{\frac{\partial F}{\partial x} = \frac{dF}{du}\frac{\partial u}{\partial x}}$$

Here $F(u) = \cos u$ and $u = f$, so get $\dfrac{\partial g}{\partial x} = -(3x^2y)\sin(x^3y + y^2)$.

Product rule:

$$\boxed{\frac{\partial(fg)}{\partial x}(x_0, y_0) = g(x_0, y_0)\frac{\partial f}{\partial x} + f(x_0, y_0)\frac{\partial g}{\partial x}}$$