MATH 103A

November 12, 2010

Exam 2

Name: Solutions - blue version

PID: ________________

There are 6 pages and 5 questions, for a total of 100 points and 20 bonus points.

No notes, no calculators, no books.

Please turn off and put away all electronic devices.
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question	Points	Score
1	20	
2	20	
3	30	
4	10	
5	20	
Total:	100	

Question	Bonus Points	Score
3	10	
4	10	
Total:	20	

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1. (20 points) Suppose that f is an automorphism of Z_{10} and that $f(3) = 1$. Determine a formula for f.

Find $f(1)$. $7 \cdot 3 = 21 = 1$ in $\mathbb{Z}_{10}$ so $f(7 \cdot 3) = 7f(3) = 7$.

whence $f(k) = 7k$ by the homomorphism property.

2. (a) (5 points) How many distinct left cosets of $H = \{\varepsilon, (123), (132)\}$ are there in the group

$$A_4 = \{\varepsilon, (123), (132), (124), (142), (234), (243), (134), (143), (12)(34), (13)(24), (14)(23)\}?$$

(b) (15 points) Determine them and list them (without repetitions).

(a) $[G:H] = \frac{|G|}{|H|} = \frac{12}{3} = 4$

(b) $H = \{e, (123), (132)\}$

$(124)H = \{(124), (32)(14), (341)\}$

$(142)H = \{(142), (342), (24)(31)\}$

$(143)H = \{(143), (34)(12), (243)\}$

3. (a) (10 points) What are the possible orders of the elements of S_4?

   (b) (10 points) What are the possible orders of the elements of S_5?

   (c) (10 points) How many elements of order 2 are there in $S_3 \oplus S_3$?

   (d) (10 points (bonus)) How many elements of order 2 are there in $S_4 \oplus S_5$?

See white version.

4. (a) (10 points) Let G be a finite group, H be a subgroup of G, and K be a subgroup of H ($K \subseteq H \subseteq G$). If there are four left cosets of K in H and five left cosets of H in G, how many left cosets of K are there in G?

   (b) (10 points (bonus)) Prove your result holds even when $|G| = \infty$.

(a) $[G:K] = \frac{|G|}{|K|} = \frac{|G|}{|H|}\frac{|H|}{|K|} = 5 \cdot 4 = 20$

(b) See Pink / white

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5. (20 points) Let p and q be distinct odd prime numbers. Prove that a cyclic group cannot be isomorphic to $U(pq)$. You can use the fact that $U(s)$ is cyclic for any prime s.

$U(pq) \cong U(p) \oplus U(q)$ since $(p,q)=1$ as they are distinct.

Also $U(p) \cong \mathbb{Z}_{p-1}$ And $U(q) \cong \mathbb{Z}_{q-1}$ As p, q are odd

we have that $2 \mid p-1$ and $2 \mid q-1$ so $(p-1, q-1) \neq 1$

And so $\mathbb{Z}_{p-1} \oplus \mathbb{Z}_{q-1}$ is <u>not</u> cyclic.