

Exam 1

Name: SOLUTIONS - green version

PID: _____

There are 7 pages and 6 questions, for a total of 120 points.

No notes, no calculators, no books.

Please turn off and put away all electronic devices.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Unless otherwise stated, show all your work for full credit. To maximize credit, cross out incorrect work.

Good luck! ☺

Question	Points	Score
1	15	
2	20	
3	20	
4	20	
5	35	
6	10	
Total:	120	

1. (15 points) List all the cyclic subgroups of $U(20)$.

$$U(20) = \{1, 3, 7, 9, 11, 13, 17, 19\}$$

$$\langle 1 \rangle = \{1\}$$

$$\langle 3 \rangle = \{1, 3, 9, 7\}$$

$$\langle 7 \rangle = \{1, 7, 9, 3\} = \langle 3 \rangle$$

$$\langle 9 \rangle = \{1, 9\}$$

$$\langle 11 \rangle = \{1, 11\}$$

$$\langle 13 \rangle = \{1, 9, 17, 13\}$$

$$\langle 17 \rangle = \{1, 17, 9, 13\} = \langle 13 \rangle$$

$$\langle 19 \rangle = \{1, 19\}$$

Subgroups are $\{1\} = \langle 1 \rangle$

$$\{1, 3, 7, 9\} = \langle 3 \rangle = \langle 7 \rangle$$

$$\{1, 9\} = \langle 9 \rangle$$

$$\{1, 11\} = \langle 11 \rangle$$

$$\{1, 9, 13, 17\} = \langle 13 \rangle$$

$$\{1, 19\} = \langle 19 \rangle$$

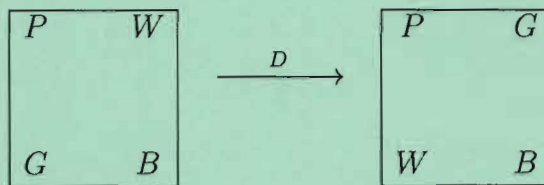
2. (a) (10 points) How many elements of order 9 does $(\mathbb{Z}_{18}, + \pmod{18})$ have? Justify your answer.
- (b) (10 points) How many elements of order 9 does \mathbb{Z}_{2010} have? Justify your answer.

(a) \mathbb{Z}_{18} cyclic of order 18
 $9 \mid 18$ $\Bigg| \Rightarrow \mathbb{Z}_{18}$ has
 $\varphi(9) = 6$
 elements of order 9

(b) \mathbb{Z}_{2010} cyclic of order 2010 $\Bigg| \Rightarrow$
 $9 \nmid 2010$

\Rightarrow there are no elements of order 9 in \mathbb{Z}_{2010}

3. (20 points) Compute the centralizer in D_4 of the flip across the main diagonal.



It might help (but you don't have to use this if you know a simpler way to solve the problem) to know that the matrix of a ~~clockwise~~ clockwise rotation by angle θ

is $R_\theta = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, and the matrix of a flip across a line that makes an angle α

with the positive x -axis is $S_\alpha = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$.

See the other versions

answer : $\{R_0, R_{180}, D, D'\}$

flip ~~across~~ across the other diagonal.

4. (20 points) Suppose that (G, \cdot) is a group and that $x^2 = e$ for all elements $x \in G$. Prove that G is abelian.

See yellow version

5. Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R}, a \neq 0 \right\}$.

- (a) (10 points) Find an element of G that acts like the identity element in G with respect to matrix multiplication (i.e. find $X \in G$ such that $AX = XA = A$ for all $A \in G$.)
- (b) (10 points) Show that G is closed under matrix multiplication (i.e. show that $AB \in G$ for any matrices $A \in G$ and $B \in G$.)
- (c) (15 points) Show that $(G, \text{matrix multiplication})$ is a group. You can assume associativity, since we know that matrix multiplication is associative in general.

see blue version

6. (10 points) Decompose into disjoint cycles the permutation

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 8 & 6 & 5 & 1 & 3 & 4 & 7 \end{bmatrix}.$$

$$(128745)(36)$$