MATH 103A – Practice Problems for the Final

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Please do not make any assumptions about the composition of the exam from this set of review problems. Do not assume that the exam questions will be exactly as the questions below, or slight modifications of them. The test problems may look completely different.problems (closed book, closed notes) then you have necessary knowledge and skills to do well on the midterm. It may be a good idea to time yourself just like in an actual exam.

Also this set is not an indication of how many problems of each type you will encounter on the exam.

1. Go over supplementary exercises for Chapters 1–4, 5–8, 9–11.
2. Write down 5 different groups of order 24, such that no two of them are isomorphic to each other. Prove that no two are isomorphic.
3. Fix some $n \geq 3$, and let $\alpha \in S_n$ be an *odd* permutation. Prove that there must exist an *even* permutation β such that $\alpha = (12)\beta$.
4. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 6 & 2 & 4 & 1 & 5 \end{bmatrix}$ be a permutation in S_7. Find α^{34}, expressing your answer in cycle notation.
5. Suppose that N is a **normal** subgroup of the symmetric group S_5. Show that if $(1, 2) \in N$ then $N = S_5$.
6. Prove that an Abelian group with two elements of order 2 must have a subgroup of order 4.
7. Suppose that H and K are subgroups of a group G. If $|H| = 15$ and $|K| = 28$, find $|H \cap K|$.
8. Explain why the function, $\varphi : \mathbb{Z}_{12} \to \mathbb{Z}_{10}$ defined by $\varphi(x) = x \bmod 10$ is **not** a group homomorphism.
9. Describe all of the homomorphisms, φ, from $\mathbb{Z}_{12} \to \mathbb{Z}_{10}$.
10. Let N be a normal subgroup of a finite group G. Prove that the order of the group element gN in G/N divides the order of g.
11. How many elements of order 6 are in $S_3 \oplus S_3$? How many cyclic subgroups of order 6 does $S_3 \oplus S_3$ have?
12. Find 11^{-1} where 11 is thought of as an element of $U(19)$.
13. Find all left cosets of $H = \{1, 11\}$ in $G = U(20)$. Find the isomorphism class of G/H.
14. Let G be a group of permutations of the set $S = \{1, 2, 3\}$ such that $\text{orb}_G(2) = \{1, 2\}$. Determine G.
15. Up to isomorphism, how many abelian groups of order 210 are there? How about order 40?
16. Compute the centralizer of the flip around the horizontal line in D_4.
17. What are the possible orders of the elements of $U(72)$?
18. Find the isomorphism class of $U(72)$.