

Titles and abstracts of talks  
Workshop “von Neumann algebras and ergodic theory”  
UCLA, September 22nd-26th, 2014

**Vadim Alekseev** (Gottingen U)

*Title:* Derivations on von Neumann algebras and  $L^2$ -cohomology

*Abstract:* Connes and Shlyakhtenko introduced  $L^2$ -Betti numbers for subalgebras of finite von Neumann algebras in hope to get a nice homological invariant connected to  $L^2$ -Betti numbers for groups. Later on, Andreas Thom introduced the cohomological approach to  $L^2$ -Betti numbers for von Neumann algebras, connecting them with derivations. In the talk, I'll present the development of the  $L^2$ -invariants for finite von Neumann algebras and explain the recent vanishing results for first continuous  $L^2$ -cohomology.

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**Rémi Boutonnet** (UCSD)

*Title:* Maximal amenable von Neumann subalgebras arising from amenable subgroups

*Abstract:* Consider a maximal amenable subgroup  $H$  in a discrete countable group  $G$ . In this talk I will give a general condition implying that the von Neumann subalgebra  $LH$  is still maximal amenable inside  $LG$ . The condition is expressed in terms of  $H$ -invariant measures on some compact  $G$ -space. As an example I will show that the subgroup of upper triangular matrices inside  $SL(n, \mathbb{Z})$  gives rise to a maximal amenable subalgebra. This talk is based on a joint work with Alessandro Carderi.

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**Lewis Bowen** (UT Austin)

*Title:* Equivalence relations acting on bundles of hyperbolic spaces

*Abstract:* Under appropriate properness and local compactness conditions, a measured equivalence relation that acts isometrically on a bundle of hyperbolic spaces has nice properties. For example, every aperiodic hyperfinite subequivalence relation is contained in a unique maximal hyperfinite subequivalence relation and every nonamenable subequivalence relation contains a treeable nonamenable subequivalence relation. I'll explain this and other nice properties. This is joint work with Sukhpreet Singh.

**Ionut Chifan** (U of Iowa)

*Title:* Some structural results for the von Neumann algebras associated with braid groups

*Abstract:* In this talk I will present some recent rigidity results for the von Neumann algebras associated with actions of braid groups. We will show that any free ergodic pmp action of the central quotient of the braid group with at least five strands on a probability space is virtually  $W^*$ -superrigid; this means that any such action can be completely reconstructed from its von Neumann algebra. The proof uses a dichotomy theorem of Popa-Vaes for normalizers inside crossed products by free groups in combination with a OE-superrigidity theorem of Kida for actions of mapping class groups. Other structural results such as primeness or unique tensor factorisations for the von Neumann algebras associated with braid groups will also be discussed. This is based on an initial joint work with A. Ioana and Y. Kida and a subsequent joint work with S. Pant.

**Benjamin Hayes** (Vanderbilt U)

*Title:* Fuglede-Kadison Determinants and Sofic Entropy

*Abstract:* In this talk we are concerned with invariants of an algebraic action. An algebraic action is an action of a countable discrete group  $G$  on a compact, metrizable, abelian group  $X$  by automorphisms. We typically forget the algebraic structure of  $X$  and so when we talk about invariants we mean as a pmp action or an action on a compact metrizable space by homeomorphisms. Our results will connect the entropy of an action of a sofic group  $G$  on the Pontryagin dual of a finitely presented  $Z(G)$ -module with Fuglede-Kadison determinants under essentially minimal assumptions. Our results generalize and build on previous results due to Bowen, Deninger-Schmidt, Kerr-Li, Li-Thom, Lind-Schmidt-Ward and others. In particular, they are a partial generalization of results due to Li-Thom from the amenable case to the sofic case. Time permitting, we will sketch the details of the proof. The proof appears to be the first in the subject to avoid a nontrivial determinant approximation.

**Dan Hoff** (UCSD)

*Title:* Von Neumann Algebras of Equivalence Relations with Nontrivial One-Cohomology

*Abstract:* A natural question in the classification of von Neumann algebras asks which type  $\text{II}_1$  factors are prime, i.e. which  $\text{II}_1$  factors cannot be written as the tensor product of two diffuse von Neumann subalgebras. Peterson showed that the group von Neumann algebra  $L(\Gamma)$  is prime for any countable group  $\Gamma$  which admits an unbounded 1-cocycle into the left regular representation. In this talk, we focus on the case of the von Neumann algebra  $L(\mathcal{R})$  for a countable ergodic pmp equivalence relation  $\mathcal{R}$  and give an analogous result in this case:  $L(\mathcal{R})$  is prime for any nonamenable  $\mathcal{R}$  which admits an unbounded 1-cocycle into a mixing

orthogonal representation weakly contained in the regular representation. We outline a proof based on Popa's powerful deformation/rigidity theory.

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**Yusuke Isono** (Kyoto)

*Title:* Free independence in ultraproduct von Neumann algebras and applications

*Abstract:* We generalize Popa's free independence result for ultra products of  $II_1$  factors to the framework of type III factors with large centralizer algebras. Then we give two applications. First, we give a direct proof of stability under free product of QWEP for von Neumann algebras. Second, we give a new class of inclusions of von Neumann algebras with relative Dixmier property. This is a joint work with C. Houdayer.

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**Alekos Kechris** (Caltech)

*Title:* Descriptive graph combinatorics and ergodic theory

*Abstract:* Descriptive graph combinatorics studies definable graphs, usually Borel or analytic, on Polish spaces and investigates how combinatorial concepts, such as colorings and matchings, behave under definability or regularity (in the measure theoretic or topological sense) constraints. This subject has some interesting connections with ergodic theory that we will discuss in this talk.

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**David Kerr** (Texas AM)

*Title:* Dynamics and dimension

*Abstract:* The notion of dimension within the context of amenable groups and their actions has become an important tool both in the classification theory of nuclear  $C^*$ -algebras and in the study of the relation between  $K$ -theory and asymptotic geometry. I will present a combinatorial perspective on this dimension theory and speculate about its applications in ergodic theory and operator algebras.

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**Yoshikata Kida** (Kyoto)

*Title:* Stable orbit equivalence relations and relative property (T)

*Abstract:* We show that if a group  $G$  has a central subgroup  $C$  such that the pair  $(G, C)$  does not have property (T), then  $G$  admits an ergodic, free and p.m.p. action whose orbit

equivalence relation is stable. We also show that any central extension of a treeable group has the Haagerup property, and therefore if its center is infinite, then it admits such an action.

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**Brian Leary** (UCLA)

*Title:* On maximal amenable subalgebras of amalgamated free product von Neumann algebras

*Abstract:* In the years since Popa first showed that generator subalgebras in free group factors are maximal amenable, there has been much recent work giving new maximal amenability results. In this talk, I will use Popa's argument to show that an amenable diffuse finite von Neumann  $N_1$  is maximal amenable inside the amalgamated free product  $N_1 *_B N_2$  under certain restrictions on the common subalgebra  $B$ .

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**Hanfeng Li** (SUNY Buffalo)

*Title:* Ergodicity of principal algebraic actions

*Abstract:* For a countable group  $G$  and an element  $f$  of the integral group ring  $\mathbb{Z}G$  of  $G$ , one may consider the  $G$ -action on the Pontryagin dual of  $\mathbb{Z}G/\mathbb{Z}Gf$ . I will discuss when such a principal algebraic action is ergodic with respect to the Haar measure. This is joint work with Jesse Peterson and Klaus Schmidt.

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**François Le Maître** (ENS Lyon)

*Title:* More Polish full groups

*Abstract:* Full groups were introduced in Dye's visionary paper of 1959 as subgroups of  $\text{Aut}(X, \mu)$  stable under cutting and gluing their elements along a countable partition of the probability space  $(X, \mu)$ . However, the focus has since then been rather on full groups generated by countable groups, which are Polish for the uniform topology. This situation is justified by the very nice interplay between these full groups and the ergodic theory of countable measure preserving equivalence relations. On the other hand, note that the group  $\text{Aut}(X, \mu)$  itself is a full group, Polish for the weak topology, and its topological properties are still an important subject of investigation. In a work in progress with A. Carderi, we unveil Polish full groups of a new kind, whose topology is intermediate between the uniform and the weak topology. They arise as full groups of equivalence relations generated by the Borel action of a Polish group on  $(X, \mu)$ , and we will discuss some of their topological properties such as amenability or topological rank.

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**Andrew Marks** (Caltech)

*Title:* One ended subforests of Borel graphs

*Abstract:* We investigate the problem of when a measure-preserving Borel graph on a standard probability space has a Borel a.e. one-ended subforest. We then discuss some applications to measurable graph colorings and the strong treeability of groups with planar Cayley graphs. This is joint work with Clinton Conley and Robin Tucker-Drob.

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**Niels Meesschaert** (K.U. Leuven)

*Title:* Baumslag-Solitar groups and  $W^*$ -equivalence rigidity

*Abstract:* For all  $n, m \in \mathbb{Z} \setminus \{0\}$ , the *Baumslag-Solitar group*  $BS(n, m)$  is defined by the presentation

$$BS(n, m) := \langle a, b \mid ba^nb^{-1} = a^m \rangle .$$

These groups were introduced by Baumslag and Solitar to provide the first examples of finitely presented non-Hopfian groups. We examine both the group von Neumann algebra of the Baumslag-Solitar groups as the group measure space construction of some of their actions. In the case of the group von Neumann algebra, the rational number  $|n/m|$  is an invariant. More precisely, if  $L(BS(n, m))$  is isomorphic with  $L(BS(n', m'))$ , then  $|n'/m'| = |n/m|^{\pm 1}$ . In the case of the group measure space construction, we get that  $\{|n|, |m|\}$  is an invariant whenever  $\langle a \rangle$  acts aperiodically. More precisely, let  $BS(n_1, m_1) \curvearrowright X_1$  and  $BS(n_2, m_2) \curvearrowright X_2$  be two probability measure preserving, essentially free, ergodic actions of nonamenable Baumslag-Solitar groups whose canonical almost normal abelian subgroups act aperiodically, if  $L^\infty(X_1) \rtimes BS(n_1, m_1) \cong L^\infty(X_2) \rtimes BS(n_2, m_2)$ , then  $\{|n_1|, |m_1|\} = \{|n_2|, |m_2|\}$ .

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**Brent Nelson** (UCLA)

*Title:* Free monotone transport without a trace

*Abstract:* Classical transport is a map  $T : X \rightarrow Z$  between probability spaces  $(X, \mu)$  and  $(Z, \nu)$  such that  $T_*\mu = \nu$ . Consequently,  $f \mapsto f \circ T$  provides an integral preserving embedding of  $L^\infty(Z, \nu)$  into  $L^\infty(X, \mu)$ . Free transport extends this idea to non-commutative probability spaces (i.e. pairs  $(A, \varphi)$  of von Neumann/  $C^*$ -algebras and states) to produces embeddings and even isomorphisms between non-commutative probability spaces. In this talk, we will discuss how to construct non-tracial transport by solving a non-commutative differential equation known as the Schwinger-Dyson equation and, time permitting, applications to  $q$ -deformed Araki-Woods algebras and finite depth subfactor planar algebras.

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**Narutaka Ozawa** (RIMS Kyoto)

*Title:* Noncommutative real algebraic geometry of Kazhdan's property (T)

*Abstract:* I will start with a gentle introduction to the emerging subject of “noncommutative real algebraic geometry,” a subject which deals with equations and inequalities in noncommutative algebra over the reals, with the help of analytic tools such as representation theory and operator algebras. I will then present a surprisingly simple proof that a group  $G$  has Kazhdan's property (T) if and only if a certain inequality in the group algebra  $\mathbf{R}[G]$  is satisfied. Very recently, Netzer and Thom used a computer to verify this inequality for  $\mathrm{SL}(3, \mathbf{Z})$ , thus giving a new proof of property (T) for  $\mathrm{SL}(3, \mathbf{Z})$  with a much better estimate of the Kazhdan constant than the previously known.

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**Dave Penneys** (UCLA)

*Title:* Classifying small index subfactors

*Abstract:* Recently, subfactor standard invariants have been completely classified up to index 5. A conjecture of Morrison-Peters states that there are exactly 2 non-trivial standard invariants in the index range  $(5, 3 + \sqrt{5})$ . Morrison and I conjecture in recent joint work that there are exactly 13 non-trivial standard invariants at index  $3 + \sqrt{5}$ . I'll report on current progress in joint work with Afzaly and Morrison toward proving these conjectures.

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**Jesse Peterson** (Vanderbilt U)

*Title:* Character rigidity for lattices in higher-rank simple Lie groups

*Abstract:* Inspired by the rigidity results of Margulis and Zimmer, Connes conjectured in the early 80's that if  $\Gamma$  is a irreducible property (T) lattice in a higher-rank semi-simple Lie group  $G$  with no compact factors and trivial center, then the only unitary representations of  $\Gamma$  which generate  $\mathrm{II}_1$  factors are multiples of the left-regular representation. Partial results to this question have been obtained recently by Bekka, Creutz, Thom, and the speaker. In this talk, which is based on work in progress, we present the general (positive) solution to this conjecture.

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**Sorin Popa** (UCLA)

*Title:* Towards a good cohomology theory for  $\mathrm{II}_1$  factors

*Abstract:* I will comment on the recent efforts to find a “good” 1-cohomology theory for  $\mathrm{II}_1$  factors  $M$ , i.e. one that does not always vanish and that can detect properties of  $\mathrm{II}_1$  factors

such as primeness, absence of Cartan subalgebras, infinite generation, etc. Ideally, such a theory should be calculable and in the case of group  $\text{II}_1$  factors  $M = L(\Gamma)$  it should reflect cohomology properties of the group  $\Gamma$ .

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**Hiroki Sako** (Niigata U)

*Title:* Coarse amenability for discrete metric spaces

*Abstract:* I will talk about Yus property A, which is an amenability-type condition for discrete metric spaces. Amenability was originally defined for discrete groups and characterized by a geometric property called Folner condition. The geometric condition is not so interesting in discrete metric spaces, but its modification called ‘property A is important in operator K-theory. Property A is characterized by several other properties, some are geometric, others are analytic. I will explain an equivalent condition called ‘operator norm localization property and its conclusions.

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**Mikael De La Salle** (ENS Lyon)

*Title:* Rigidity results for simple Lie groups of very high rank

*Abstract:* I will discuss two kinds of rigidity results that might hold for every simple Lie group  $G$  of real rank greater than equal to two, but that Tim de Laat and myself can only prove for simple Lie groups of very high rank. One result is a version of property (T) with respect to some Banach spaces. The other deals with the approximation properties for the non-commutative  $L_p$  spaces of the von Neumann algebra of lattices in  $G$ . The rank depends on the Banach space or the value of  $p$ .

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**Dima Shlyakhtenko** (UCLA)

*Title:* Free transport and regularity

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**Thomas Sinclair** (UCLA)

*Title:* Existentially closed  $C^*$ -algebras

*Abstract:* A  $C^*$ -algebra  $A$  is said to be existentially closed if, roughly, every set of equations involving norms of noncommutative  $*$ -polynomials which has a solution in  $B(H)$  has a sequence of approximate solutions in  $A$ . A basic result in continuous logic shows that every

separable  $C^*$ -algebra is contained in a separable, existentially closed  $C^*$ -algebra. In this talk I will survey some basic properties of existentially closed  $C^*$ -algebras. In particular I will describe how existential closure is deeply connected to several open problems in  $C^*$ -algebras such as Kirchberg's problem on whether every separable  $C^*$ -algebra embeds in an ultrapower of the Cuntz algebra  $\mathcal{O}_2$ , as well as Kirchberg's  $C^*$ -algebraic reformulation of Connes' embedding problem. No knowledge of continuous logic will be assumed. This talk is based on joint work with Isaac Goldbring.

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**Masamichi Takesaki** (UCLA)

*Title:* Similarity Entropy of a Pair of Normal States on a Factor

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**Robin Tucker-Drob** (Rutgers)

*Title:* Invariant means and the structure of inner amenable groups

*Abstract:* We study actions of countable discrete groups which are amenable in the sense that there exists a mean on  $X$  which is invariant under the action of  $G$ . Assuming that  $G$  is nonamenable, we obtain structural results for the stabilizer subgroups of amenable actions which allow us to relate the first  $\ell^2$ -Betti number of  $G$  with that of the stabilizer subgroups. An analogous relationship is also shown to hold for cost. The particular case of an atomless mean for the conjugation action of  $G$  corresponds to inner amenability. We show that inner amenable groups have cost 1 and moreover they have fixed price. We establish  $U_{\text{fin}}$  cocycle superrigidity for the Bernoulli shift of any nonamenable inner amenable group. In addition, we provide a concrete structure theorem for inner amenable linear groups over an arbitrary field. We also completely characterize linear groups which are stable in the sense of Jones and Schmidt.

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**Stefaan Vaes** (K.U. Leuven)

*Title:* Locally compact groups, cross section equivalence relations, and von Neumann algebras

*Abstract:* Given a measure preserving action of a locally compact unimodular group  $G$  on a probability space  $X$ , the restriction of the orbit equivalence relation to an appropriately chosen subset  $Y$  of  $X$  has countable orbits and gives rise to a  $\text{II}_1$  factor  $M$ . As a first application, I present the definition and basic properties of  $L^2$ -Betti numbers for locally compact unimodular groups. Secondly, I will explain that if  $G$  is a finite center simple Lie group of rank one, then  $M$  has a unique Cartan subalgebra. In specific examples, these  $M$  cannot be realized by a group measure space construction.

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**Peter Verraedt** (K.U. Leuven)

*Title:* Classification of type III Bernoulli crossed products

*Abstract:* Crossed products with noncommutative Bernoulli actions were introduced by Connes as the first examples of full factors of type III. In this talk, I present a complete classification of the factors  $(P, \phi)^{\mathbb{F}_n} \rtimes \mathbb{F}_n$ , where  $\mathbb{F}_n$  is the free group and  $P$  is an amenable factor with an almost periodic state  $\phi$ . I show that these factors are completely classified by the rank  $n$  of the free group  $\mathbb{F}_n$  and Connes's Sd-invariant. This is joint work with Stefaan Vaes.

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