3.5 #8

$y'' + y = 3\sin 2t + t\cos 2t$

Complementary Part

$y'' + y = 0$ $\qquad y_c = c_1 \cos t + c_2 \sin t$

$r^2 + 1 = 0$

$r = \pm i$

Particular Part

$Y_1 = A\sin(2t) + B\cos(2t)$

$Y_2 = (At + B)\cos 2t + (Ct + D)\sin 2t = At\cos 2t + B\cos 2t + Ct\sin 2t + D\sin 2t$

First Part: $g_1(t) = 3\sin 2t$

$Y_1' = 2A\cos(2t) - 2B\sin(2t)$

$Y_1'' = -4A\sin(2t) - 4B\cos(2t)$

$Y_1'' + Y_1 = \left[-4A\sin(2t) - 4B\cos(2t)\right] + \left[A\sin(2t) + B\cos(2t)\right] = 3\sin(2t)$

$-3A\sin 2t - 3B\cos(2t) = 3\sin(2t)$

$A = -1 \qquad B = 0$

$Y_1 = -\sin(2t)$

Second Part: $g_2(t) = t\cos(2t)$

$Y_2' = A\cos 2t - 2At\sin 2t - 2B\sin 2t + C\sin 2t + 2Ct\cos 2t + 2D\cos 2t$

$Y_2'' = -2A\sin 2t - 2A\sin 2t - 4At\cos 2t - 4B\cos 2t + 2C\cos 2t$
$+ 2C\cos 2t - 4Ct\sin 2t - 4D\sin 2t$

$= (-4A - 4D)\sin 2t + (-4B + 4C)\cos 2t - 4At\cos 2t - 4Ct\sin 2t$

$Y_2'' + Y_2 = (-4A - 4D + D)\sin 2t + (-4B + 4C + B)\cos 2t + (-4A + A)t\cos 2t + (-4C + C)t\sin 2t$

$= (-4A - 3D)\sin 2t + (-3B + 4C)\cos 2t - 3At\cos 2t - 3Ct\sin 2t$

$= t\cos(2t)$

$\underbrace{-4A - 3D = 0}_{D = -\frac{4A}{3} = \frac{4}{9}} \qquad \underbrace{-3B + 4C = 0}_{B = 0} \qquad \underbrace{-3A = 1}_{A = -\frac{1}{3}} \qquad \underbrace{-3C = 0}_{C = 0}$

$Y_2 = -\frac{1}{3}t\cos 2t + \frac{4}{9}\sin 2t$

General Solution: $\overbrace{y = c_1\cos t + c_2\sin t - \frac{5}{9}\sin 2t - \frac{1}{3}t\cos 2t}^{Y_1 + Y_2}$