Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 20D.
Midterm Exam 1
April 24, 2009

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	8	
2	6	
3	6	
4	6	
Σ	26	

1. A college graduate with no initial capital invests m dollars per year at an annual rate of return of r. Assume that investments are made continuously and that the return is compounded continuously.

   (a) (2 points) Write a differential equation and initial condition for the amount $A(t)$ of the investment at time t (in years).

   (b) (4 points) Solve the initial value problem to find amount $A(t)$ of the investment at any time t.

   (c) (2 points) If $r = 4\%$, determine m so that $1 million will be available for retirement in 40 years.

2. (6 points) Solve the initial value problem

$$\begin{cases} y' - \tan(t)\, y = 4t^3 \sec(t), \\ y(0) = 1 \end{cases}$$

(Hint: Recall that $\int \tan(t)\, dt = -\ln|\cos(t)| + C$.)

3. Consider the autonomous differential equation $\dfrac{dy}{dt} = y(y-2)$.

(a) (2 points) Identify the equilibrium solutions.

(b) (4 points) Using the axes below, sketch $f(y) = y(y-2)$ versus y and determine which, if any, of the equilibrium solutions are asymptotically stable and which, if any, are unstable.

f(y)

y
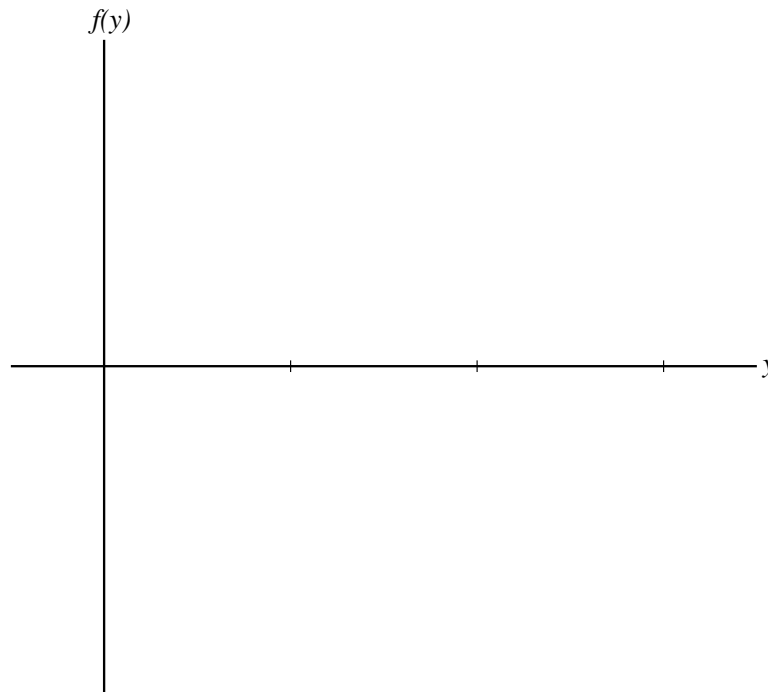
4. (a) (2 points) Verify that $(3x+4y)+(4x+5y)y' = 0$ is an exact differential equation.

(b) (4 points) Find the general solution to the differential equation.