



University of California, San Diego
Department of Mathematics

Instructions

1. Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use one page of notes, but no books or other assistance during this exam.
4. Write your solutions clearly in your Blue Book
 - (a) Carefully indicate the number and letter of each question.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start a new answer on a new page.
5. Show all of your work; no credit will be given for unsupported answers.

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. Consider the following initial value problem:

$$y' + \frac{1}{4}y = \frac{t}{4} + e^{\frac{3t}{4}}, \quad y(0) = 0.$$

- (a) (1 point) Is the differential equation linear or nonlinear?
- (b) (6 points) Solve the initial value problem.

2. Consider the following differential equation:

$$\frac{dy}{dt} = y(1 - y).$$

- (a) (1 point) Is the differential equation linear or nonlinear?
- (b) (4 points) Find the equilibrium solutions for the differential equation and identify each one as being asymptotically stable, unstable, or semistable.
- (c) (1 point) Suppose $y(0) = 1/2$. To what does y converge (if anything) as $t \rightarrow \infty$?
- (d) (6 points) Solve the initial value problem

$$\frac{dy}{dt} = y(1 - y), \quad y(0) = \frac{1}{2}.$$

(Leave your answer in *implicit* form.)

(Please turn over.)

This exam is worth 60 points.

3. (7 points) Find the general solution to the second order differential equation:

$$y'' + \frac{1}{4}y = \cos(t/2).$$

4. (4 points) Consider the differential equation

$$t^2y'' + 3ty' + y = 0, \quad t > 0.$$

Given that $y_1 = \frac{1}{t}$ is a solution to the differential equation, use the method of Reduction of Order to find a second solution.

5. (6 points) Find the general solution of the linear system

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}.$$

6. (6 points) Find the general solution of the linear system

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}.$$

7. Consider the following differential equation:

$$y'' + xy' + y = 0.$$

- (a) (4 points) Find the recurrence relation for the power series solution about the point $x_0 = 0$.
(b) (4 points) Write the first four nonzero terms for each of two linearly independent power series solutions.
(c) (1 point) Verify that the two solutions you found in part (b) form a fundamental set of solutions.
8. (a) (3 points) Use the definition of the Laplace transform to compute $\mathcal{L}\{t\}$.
(b) (5 points) Use the fact that

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}, \quad s > a, \quad n = 0, 1, 2, \dots$$

to solve the initial value problem

$$y'' + 2y' + y = e^{-t}; \quad y(0) = 0, \quad y'(0) = 0.$$

(Note: If you cannot solve this problem using Laplace transforms, you may use an alternate method to solve the problem for half credit.)

This exam is worth 60 points.