University of California, San Diego
Department of Mathematics

Instructions

1. Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use one page of notes, but no books or other assistance during this exam.
4. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start a new answer on a new page.
5. Show all of your work; no credit will be given for unsupported answers.

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0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (8 points) Solve the initial value problem:

$$\frac{dy}{dx} + 2xy^2 = 0, \qquad y(2) = \frac{1}{5}.$$

2. Consider the following differential equation:

$$\frac{dy}{dt} = e^y(10 - 3y - y^2).$$

   (a) (6 points) Find all equilibrium solutions, draw a phase line, and identify each equilibrium solution as stable, unstable, or semistable.

   (b) (1 point) Suppose $y = \phi(t)$ is a solution to the differential equation that satisfies the initial condition $y(0) = -4$. What is $\lim_{t \to \infty} \phi(t)$?

3. (6 points) Find the general solution to the differential equation:

$$t^2 \frac{dy}{dt} + t(t+2)y = e^t, \quad t > 0.$$

4. (a) (2 points) Determine if the following differential equation is exact. (You do not need to solve the differential equation.)

$$x - y^3 + y^2 \sin x = (3xy^2 + 2y \cos x)\, y'.$$

   (b) (6 points) The following differential equation is exact. Find an implicit solution:

$$\left(x^2 y^3 - \frac{1}{1+x^2}\right) dx + (x^3 y^2 + \sin y)\, dy = 0.$$

(This exam is worth 30 points.)