



*University of California, San Diego*  
*Department of Mathematics*

**Instructions**

- (a) Write your *Name, PID, Section*, and the *Version* of your exam on the front of your Blue Book.
- (b) No calculators or other electronic devices are allowed during this exam.
- (c) You may use one page of notes, but no books or other assistance during this exam.
- (d) Read each question carefully, and answer each question completely.
- (e) Write your solutions clearly in your Blue Book
  - (a) Carefully indicate the number and letter of each question.
  - (b) Present your answers in the same order they appear in the exam.
  - (c) Start each question on a new page.
- (d) Show all of your work; no credit will be given for unsupported answers.
- (e) NOTE: This exam has TWO sides and TEN questions.

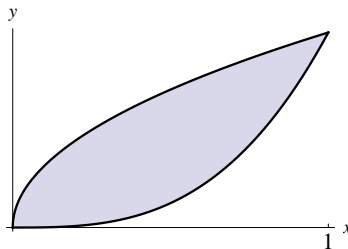
1. (10 points) Compute the definite integral:

$$\int_0^1 \frac{x e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}} dx$$

2. (10 points) Compute the indefinite integral:

$$\int x^2 \sin(3x) dx$$

3. (10 points) The shaded region below is bounded by  $y = x^2$  and  $y = \sqrt[3]{x}$ . Calculate the volume of the solid formed by rotating the region about the  $y$ -axis.



4. (10 points) Compute the following indefinite integral using complex exponentials:

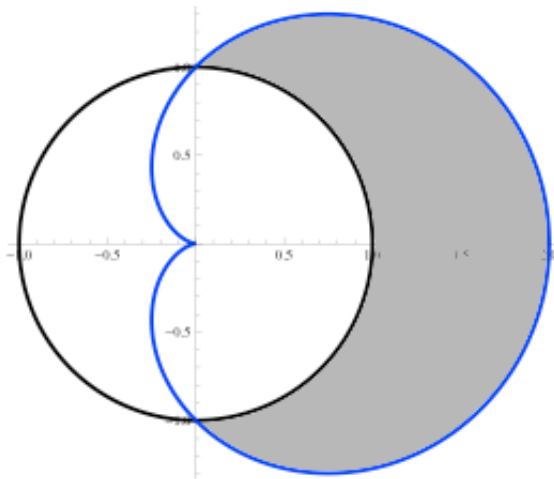
$$\int e^{2x} \cos(x) \sin(x) dx.$$

*You need not simplify the result and may leave it in complex exponential form.*

5. (10 points) Use the method of partial fractions to compute the indefinite integral:

$$\int \frac{x}{(x-1)(x^2+1)} dx$$

6. (10 points) The two polar curves below are the circle  $r = 1$  and the cardioid  $r = 1 + \cos(\theta)$ . Calculate the area of the shaded region.



7. (10 points) Compute the following definite integral:

$$\int_0^3 \frac{x^2}{\sqrt{36 - x^2}} dx$$

8. (a) (5 points) Determine if the following series converges or diverges. State which test you used and make sure to verify all conditions required by that test. **You do not need to compute the value of the series.**

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

- (b) (5 points) Explain why you can use (a) to determine whether or not the following improper integral converges or diverges?

$$\int_1^{\infty} \sin\left(\frac{1}{x^2}\right) dx.$$

9. (10 points) Determine if the following series converges absolutely, conditionally, or diverges. State which test you used and make sure to verify all conditions required by that test. **You do not need to compute the value of the series.**

$$\sum_{n=4}^{\infty} (-1)^n \frac{\ln n}{n}$$

10. (10 points) Find the interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{2n}{3^n} (x - 5)^n$$

(This exam is worth 100 points.)