University of California, San Diego
Department of Mathematics

Name: _____ PID: _____

1. Write your *NAME* on every page and your *PID* in the space provided above.
2. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
3. You may use one page of handwritten notes, but no books or other assistance during this exam.
4. Write your solutions clearly in the spaces provided.
5. Show all of your work; no credit will be given for unsupported answers.

DO NOT TURN PAGE UNTIL INSTRUCTED TO DO SO

(1 points)   0. **(Question Zero)** Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points)   1. Use the linear approximation of $f(x) = \sin\left(1 - e^{3x} + 10x\right)$ at $x = 0$ to approximate $f(1/10)$. Simplify your answer completely. (Your answer should be in the form of a fraction or a decimal.)

(6 points)   2. Find the slope of the tangent line at the point $(1, 1)$ for the curve given by the equation

$$\ln(xy) + y^2 e^x = e.$$

(4 points)   3. Evaluate $\displaystyle \int \frac{x^{2016} + x - x^{-2016}}{x^2} \, dx$

(8 points) 4. (a) Evaluate $\displaystyle\int_0^1 \left(\sin(2x) + x^3 \right) dx$

(b) Evaluate $\displaystyle\int_{-3}^{\pi/2} f(x)\, dx$, where $f(x) = \begin{cases} \cos(x) & \text{if } x \geq 0 \\ 5 & \text{if } x < 0 \end{cases}$

(6 points) 5. Find the absolute maximum and absolute minimum values of the function $f(x) = x^3 e^x$ over the infinite interval $(-\infty, \infty)$, if they exist. Justify your answer.

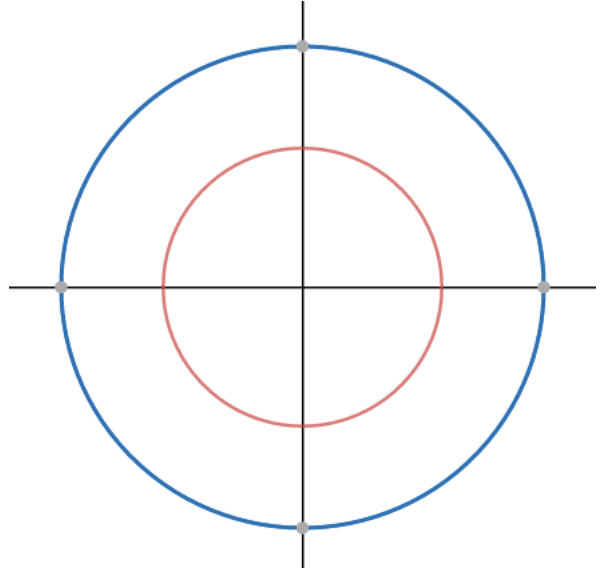(6 points) 6. Compute the limits. If a limit is infinite, state if it is $+\infty$ or $-\infty$.

(a) $\lim\limits_{x\to\infty} \left(xe^{4/x} - x\right)$

(b) $\lim\limits_{x\to 2^+} \dfrac{\ln(x-2)}{x-2}$
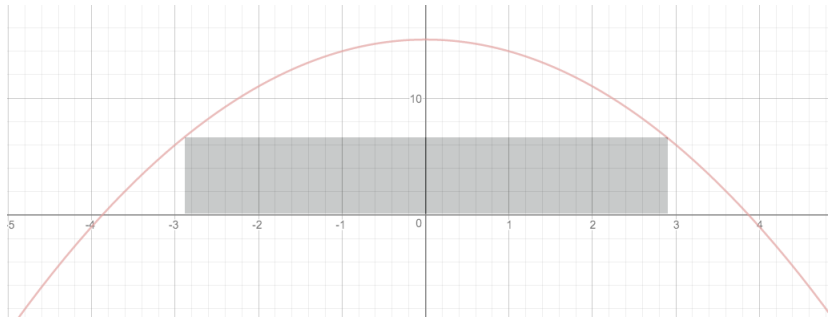
(6 points)   7. The area between two growing concentric circles is always 9π in^2. The area of the larger circle is increasing at a rate of 10π in^2/sec. How fast is the radius of the smaller circle changing when the smaller circle has area 16π in^2?

(6 points)   8. A rectangle is to be inscribed under the curve $y = 15 - x^2$ with one side along the x-axis. (One possible rectangle is shown in the picture.) What is the maximum area that such a rectangle can have?

(8 points)   9. Let $F(x) = \int_1^{3x} f(t)\,dt,$ where $f(t) = \int_t^{t^2} \sin\left(\frac{1}{u}\right)\,du.$ Compute $F'(\frac{1}{3})$ and $F''(x).$

(The function name f should not appear in your answers.)

(8 points) 10. Let $y = \left(\ln(x) \right)^{1/x^2}$. Compute (a) $\dfrac{dy}{dx}$ and (b) $\lim\limits_{x \to \infty} y$.

(7 points) 11. Let $g(x) = \displaystyle\int_0^x e^{t^2}\, dt$.

(a) Use the right-endpoint approximation with $N = 3$ intervals to estimate the value of $g(6)$. (You do not need to simplify. It is okay if the number e appears in your answer.)

(b) Over which interval or intervals is g concave down?

(8 points) 12. Suppose g is a function such that $\sin(2x) \le g(x) \le 5x^2 + 2x$ for all x. Find the value of $g'(0)$.
(*Hint:* Use the limit definition of $g'(0)$.)