University of California, San Diego
Department of Mathematics

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Write your solutions clearly in your Blue Book.**
   - **(a) Carefully indicate the number and letter of each question and question part.**
   - **(b) Present your answers in the same order they appear in the exam.**
   - **(c) Start each problem on a new page.**
5. **Show all of your work. No credit will be given for unsupported answers (even if correct).**
6. **Turn in your exam paper with your Blue Book.**

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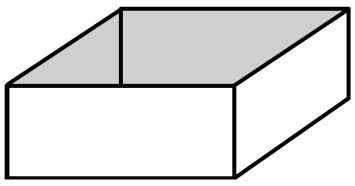
0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (10 points) Evaluate the following limits or state that they do not exist:
   (a) $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$
   (b) $\lim_{x\to 0} \dfrac{e^{3x^2}-1}{x^2}$
   (c) $\lim_{x\to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$

2. (6 points) Find the equation of the tangent line to the curve $x^{1/3} + y^{1/3} = 2xy$ at the point $(1, 1)$.

3. (10 points) For each of the following, find $\dfrac{dy}{dx}$. (You do not need to simplify.)
   (a) $y = \sin(4 + \ln x)$
   (b) $y = \dfrac{x^2 + \cos(x)}{2 + \tan(x^2)}$
   (c) $y = \sqrt{x + \sqrt{1 + 3x}}$

4. (10 points) A box with **open top** and **square base** has volume 32 cubic inches. Find the dimensions that minimize the surface area.



5. (6 points) Let $F(x) = \displaystyle\int_4^{x^2} \cos^2(2t-1)\,dt$.
   (a) Find $F(2)$.
   (b) Find $F'(x)$.

6. (12 points) Consider the function $f(x) = \dfrac{e^x}{x^2}$, which has derivatives given by

$$f'(x) = \frac{e^x(x-2)}{x^3} \quad \text{and} \quad f''(x) = \frac{e^x(x^2-4x+6)}{x^4}.$$

(***Note:*** These derivatives are given to you, so you do not need to compute them again.)

(a) What is the domain of f?

(b) Find all critical points of f (if any) and determine if each is a local maximum, local minimum, or neither.

(c) Find the inflection points of f (if any) and determine the intervals where f is concave up or concave down.

(d) Does this function have any vertical asymptotes?

(e) Compute the following limits:

$$\lim_{x\to-\infty} \frac{e^x}{x^2} \quad \text{and} \quad \lim_{x\to\infty} \frac{e^x}{x^2}.$$

Does the function f have any horizontal asymptotes?

7. (7 points) Compute the indefinite integral:

$$\int (16x^{15} - 2x^2 + 4x^{-1})\,dx$$

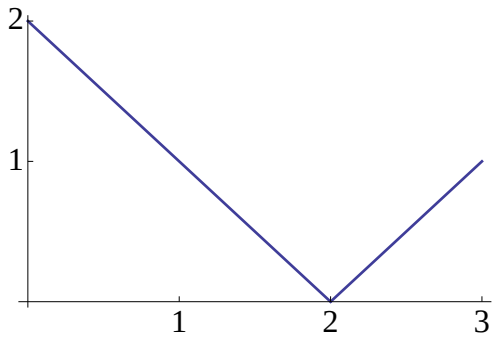
8. (7 points) Compute the definite integral:

$$\int_0^{\pi} (x^3 + \sin x + \pi)\,dx$$

9. (6 points) The function $f(x)$ is defined for x in $[0, 3]$ by the formula

$$f(x) = \begin{cases} 2 - x & \text{if} \quad 0 \le x \le 2 \\ x - 2 & \text{if} \quad 2 < x \le 3 \end{cases}$$

The graph of $y = f(x)$ is given below:



(a) Compute $\displaystyle\int_0^3 f(x)\,dx$. (You may use the graph to compute your answer, if you wish.)

(b) Compute the right-endpoint approximation R_3.