Name: _____ PID: _____

- Print your *NAME* on every page and write your *PID* in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.

(1 pt)   0. Follow the instructions on this exam and any additional instructions given during the exam.

(4 pt)   1. The following is the graph of $y = f(x)$, where $f(x)$ is a third degree polynomial.
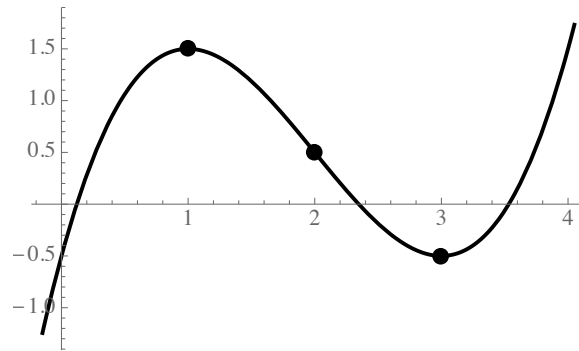
    (a) On which interval or intervals is the first derivative $f'(x)$ positive?

    (b) On which interval or intervals is the second derivative $f''(x)$ negative?

(5 pt)   2. Find the value of a and b that make the function g is continuous.

$$g(x) = \begin{cases} 1/x^2 & \text{if } x < -2 \\ ax + b & \text{if } -2 \le x \le 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

(5 pt)   3. For the curve $y = \arctan(e^x)$, find an equation for the tangent line where $x = 0$.

(5 pt)   4. Compute $f'(x)$ and $f''(x)$ if $f(x) = \ln(x^2 + 1)$. What is the domain of f?

(5 pt)   5. Let

$$f(x) = \frac{x^6}{3} - \frac{x^4}{2} + \frac{2}{3}.$$

   (a) Find the critical points for the function f and identify each as a local maximum, local minimum, or neither.

   (b) Find all points of inflection for the function f.

(5 pt)   6. In the theory of relativity, the mass of a particle with speed v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Suppose that a particle having mass 1 at rest is moving according to the position function $s(t) = t\sin(\frac{\pi t}{4})$.

(a) Compute the instantaneous velocity of the particle at time $t = 2$.

(b) Compute the instantaneous acceleration of the particle at time $t = 2$.

(c) Use parts (a) and (b) and the Chain Rule to compute $\frac{dm}{dt}$ at time $t = 2$.

(5 pt) 7. Let f be a one-to-one differentiable function such that $f(3) = 6$ and $f'(3) = \pi$. Calculate the derivatives.

(a) $g'(3)$ if $g(x) = \left(f(x)\right)^{-1}$.

(b) $h'(6)$ if $h(x) = f^{-1}(x)$.

(5 pt) 8. If $f(x) = \dfrac{\arctan(x)}{x}$ for $x \neq 0$, then what value should be assigned to $f(0)$ in order to make f a function that is continuous everywhere?

(5 pt)   9. A box with square base and no lid has volume 1. What is the minimum surface area the box can have?

(5 pt) 10. Use implicit differentiation to find the slope of the line tangent to the ellipse $\dfrac{x^2}{2} - y^2 = 4$ at the point $(4, -2)$.