68thANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 26th, 2025
on the campus of the
University of California, San Diego

PART II
4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name _____
First Last

School _____

Contest ID number _____

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and Contest ID number on **each page of the contest**.
2. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
3. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader's attention to the back side if you use it.
4. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Banquet on May 9th.

Problem 1 *Let $P_1, P_2, \ldots, P_n$ be polygons no two of which are similar. Show that there are polygons $Q_1, Q_2, \ldots, Q_n$ with Q_i similar to P_i so that for no $i \neq j$ does Q_i contain a polygon congruent to Q_j.*

Problem 2 *Let $P(X)$ be a polynomial with real coefficients such that $P(X^n) \mid P(X^{n+1})$, for all $n \in \mathbb{N}$. Prove that $P(X) = cX^k$, for some real constant c and $k \in \mathbb{N}$.*

Problem 3 *Consider a collection of $2n$ points in the plane, no three of which are collinear, and some set of line segments between them. We say that a subset of these line segments is a* pairing *if every one of these points is the endpoint of exactly one of the chosen line segments. Show that for every $k \geq 1$, there exists such an arrangement of points and line segments (for some value of $n \geq 1$) such that there are exactly k distinct pairings.*

Problem 4 *Let $a_1, a_2, a_3, \ldots$ be the sequence $2, 1, 1, 2, \ldots$ so that a_i is 1 or 2 for each i and so that the decimal number $a_n a_{n-1} \cdots a_1$ is divisible by 2^n for each $n \geq 1$. Show that the decimal $0.a_1 a_2 a_3 \ldots$ is irrational.*